Excitation of ^{229m}Th in the electron bridge via continuum, as a scattering process

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A.Ya.Dzyublik, Phys. Rev. C **102**, 024604 (2020).

^{229}**Th is a unique nucleus, having the isomeric** $3/2^+$ **level (**^{229m}**Th) with energy, lying in the ultraviolet region.**

B.Seiferle, et al., Nature, 573, 243 (2019) reported

$E_{is} = 8.28 \pm 0.17$ **eV**

$$\mathbf{T_{1/2} = 7 \pm 1 \mu s}, \qquad \mathbf{\Gamma = 10^{-10} eV}, \qquad \alpha = \frac{\mathbf{\Gamma_e}}{\mathbf{\Gamma_\gamma}} = \mathbf{10^9}$$

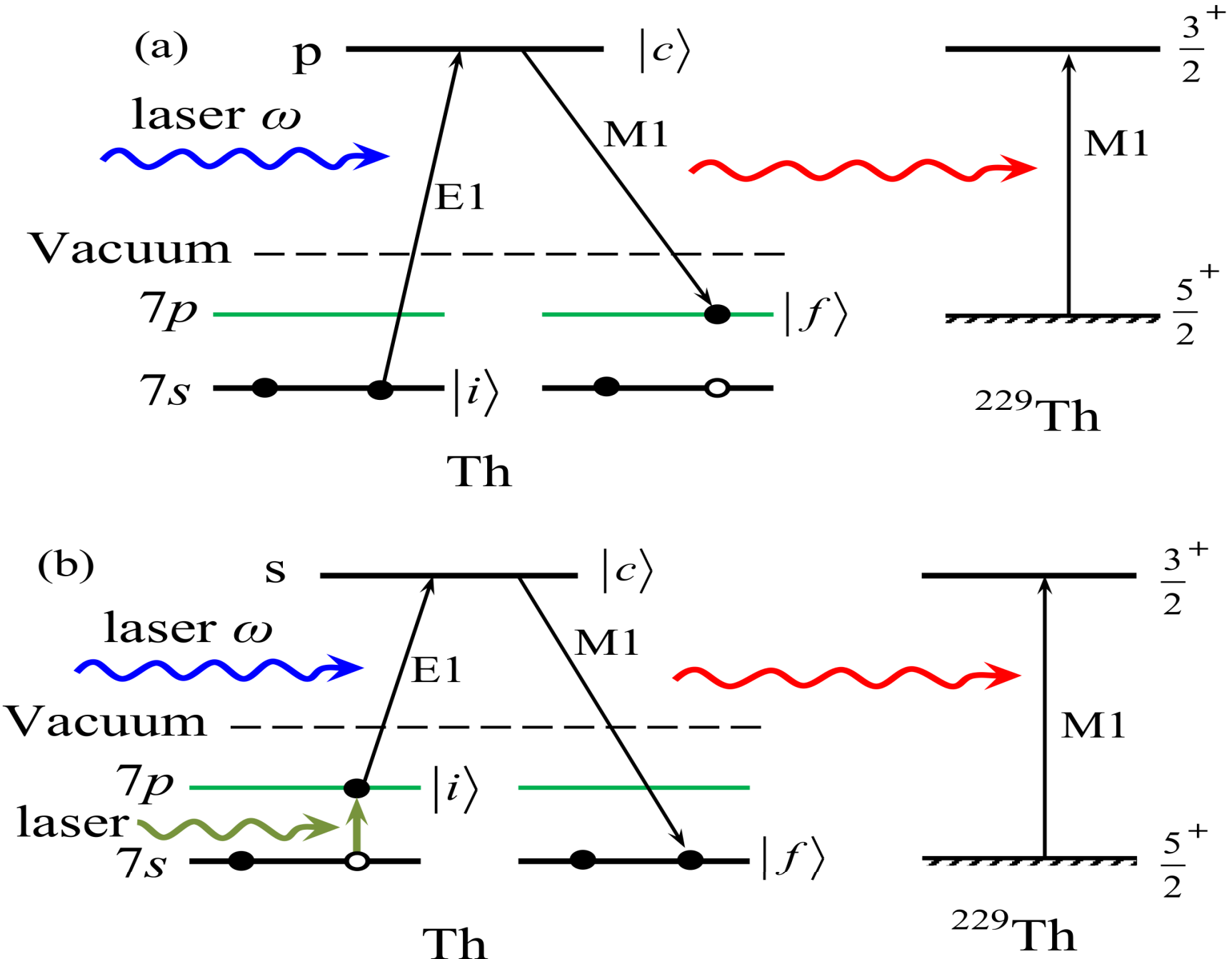
P.V.Borisyuk, et al., Phys. Rev. C **100**, 044306 (2019)

They calculated the cross section of the reaction using complex energies,

$$E_{is} \to E_{is} - i\Gamma/\hbar,$$

which lead to divergency of $\Psi_{is}(\mathbf{r})e^{-i(E_{is}-i\Gamma/2)t/\hbar}$ at $t \to -\infty$.

In addition, in $|S_{fi}|^2/T$ they ad hoc replaced T by the lifetime of the isomer $\hbar/\Gamma$.


(a)
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Th
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M1
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Vacuum
$7p$
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Th
$\frac{3}{2}^+$
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^{229}Th

The basis wave functions

The Hamiltonian of the whole system (^{229}Th + atomic electron + electromagnetic field) is written as

$$H = H_0 + V_r,$$

where the unperturbed Hamiltonian

$$H_0 = H_n + H_e + H_{\mathbf{rad}}.$$

The perturbation

$$V_r = V_r^n + V_r^e$$

is responsible for interaction of the nucleus and electron with the field,

$$V_r^{n(e)} = -\frac{1}{c}\int \mathbf{J}_{n(e)}(\mathbf{r})\mathbf{A}(\mathbf{r})d\mathbf{r},$$

where $\mathbf{J}_n(\mathbf{r})$ and $\mathbf{J}_e(\mathbf{r})$ are the electric current density operators for the nucleus and electron, respectively, $\mathbf{A}(\mathbf{r})$ the vector potential operator of the field. The operator H_e is a sum of the kinetic energy operator of the electron and the screened Coulomb field of the nucleus $V_{\mathbf{C}}(r)$.

In the initial state

$$|a\rangle = |I_g M_g\rangle \phi_{j_i m_i}(\mathbf{r}) |1_{\mathbf{k}\lambda}\rangle$$

there are the nucleus in the ground state $|I_g M_g\rangle$ with spin I_g and its projection on the quantization axis M_g, the electron with the total angular momentum j_i and its projection m_i, and one photon with the wave vector k and circular polarization $\lambda = \pm 1$.

The energy

$$E_a = \omega + \epsilon_i$$

, where ω is the energy of the incident photon, ϵ_i is the electron energy.

Having absorbed a photon, the electron passes to the continuous spectrum with the wave vector $\boldsymbol{\kappa}$, energy ε and spin projection ν. Such a first intermediate state of the system is described by the wave function

$$|c_1\rangle = |I_g M_g\rangle \psi^+_{\boldsymbol{\kappa}\nu}(\mathbf{r}) |0\rangle,$$

where $\psi^+_{\boldsymbol{\kappa}\nu}(\mathbf{r})$ describes the conversion electron, the factor $|0\rangle$ stands for the vacuum of the field.

Afterwards free electron can return to a vacant atomic level $|j_f m_f\rangle$ transferring its energy via a virtual photon to ^{229}Th, which goes to the excited isomeric level $|I_e M_e\rangle$ with $I_e^\pi = 3/2^+$. The system then passes to the second intermediate state

$$|\mathbf{c_2}\rangle = |\mathbf{I_e M_e}\rangle \phi_{\mathbf{j_f m_f}}(\mathbf{r})|\mathbf{0}\rangle.$$

The corresponding eigenvalues of H_0 are

$$E_{c_1} = \varepsilon, \qquad E_{c_2} = E_{\mathbf{is}} + \epsilon_f,$$

where ε means the energy of the conversion electron, ϵ_f is the final energy of the electron.

At the final stage of the EB process the isomer decays mainly through the conversion channel.

Let us consider now the distorted wave functions of the electron $\psi^+_{\boldsymbol{\kappa}\nu}(\mathbf{r})$ in the screened Coulomb field $V_{\rm c}(r)$. They are eigenfunctions of H_e.

Let us expand $\psi^+_{\boldsymbol{\kappa}\nu}$ in partial waves:

$$\psi^+_{\boldsymbol{\kappa}\nu}(\mathbf{r}) = \sum_{l=0}^{\infty} i^l e^{i\delta_l(\kappa)} \frac{w_l(\kappa;r)}{\kappa r} \sum_{m=-l}^{l} Y^*_{lm}(\hat{\boldsymbol{\kappa}}) Y_{lm}(\hat{\mathbf{r}}) u_{s\nu},$$

where $u_{s\nu}$ is the spin factor ($s = 1/2,\ \nu = \pm 1/2$), and $\delta_l(\kappa)$ the phase shift.

The radial functions $w_l(\kappa;r)$ satisfy the equation

$$w_l''(\kappa;r) - \left[l(l+1)/r^2 + v(r) - \kappa^2\right] w_l(\kappa;r) = 0,$$

where the reduced potential

$$v(r) = 2\mu V_{\rm c}(r)/\hbar^2.$$

Since the Hamiltonian H is invariant with respect to rotations it is more convenient to expand $\psi^+_{\boldsymbol{\kappa}\nu}(\mathbf{r})$ in terms of the generalized spherical harmonics

$$\mathscr{Y}^{m_c}_{j_c ls}(\hat{\mathbf{r}}) = \sum_{m\nu} (lsm\nu|j_c m_c) Y_{lm}(\hat{\mathbf{r}}) u_{s,\nu}.$$

where $(j_1 j_2 m_1 m_2|jm)$ are the Clebsh-Gordan coefficients.

They are eigenfunctions of the operators $\mathbf{j}^2$, $\mathbf{l}^2$ and j_z, where $\mathbf{j} = \mathbf{l} + \mathbf{s}$ is the total angular momentum operator of the electron, l and s are its orbital momentum and spin operators, respectively.

Then the wave function takes the form

$$\psi^+_{\boldsymbol{\kappa}\nu}(\mathbf{r}) = \sum_{l=0}^{\infty} \sum_{j_c m_c} \phi_{j_c l m_c}(\mathbf{r}) \mathfrak{Y}^{m_c^*}_{j_c}(ls\nu; \hat{\boldsymbol{\kappa}}),$$

depending on the products of the wave functions

$$\phi_{j_c l m_c}(\mathbf{r}) \equiv |j_c m_c\rangle = \frac{w_l(\kappa, r)}{\kappa r} \mathscr{Y}^{m_c}_{j_c ls}(\hat{\mathbf{r}}),$$

and subsidiary functions

$$\mathfrak{Y}^{m_c}_{j_c}(ls\nu; \hat{\boldsymbol{\kappa}}) = i^{-l} e^{-i\delta_l} \sum_{m=-l}^{l} (lsm\nu|j_c m_c) Y_{lm}(\hat{\boldsymbol{\kappa}}).$$

Transition matrix

The transition operator is

$$T = V + VG^{+}(E_a)V,$$

where the Green operator

$$G^{+}(E_a) = (E_a + i\eta - H)^{-1}, \qquad \eta \to +o.$$

The EB transition $a \to b$ is determined by the T matrix,

$$T_{ba} = \sum_{c_2,c_1} \langle b|R|c_2\rangle G^{+}_{c_2c_1}(E_a)\langle c_1|V_r|a\rangle,$$

where the level shift operator R is given by a series

$$R = V_r + V_r \frac{1}{E_a + i\eta - H_0} V_r + \dots$$

Here the summation over c_1 includes integration over the wave vector of the conversion electron $\boldsymbol{\kappa}$ at infinity and summation over its spin projections $\nu = \pm 1/2$. The summation over c_2 is carried over the magnetic quantum numbers M_e and m_f.

In T_{ba} the factor $\langle b|R|c_2\rangle$ is associated with the decay of the isomeric $5/2^+$ level, whose total width is defined by standard formula:

$$\Gamma_{\text{is}} = 2\pi \sum_b |\langle b|R|c_2\rangle|^2 \delta(E_b - E_a).$$

This Γ_{is} is a sum of the radiative partial width Γ_γ and the internal conversion width Γ_e.

Thus, there are two overlapping resonant levels $|c_1\rangle$ and $|c_2\rangle$, having the widths Γ_1 and Γ_2. Here $\Gamma_1/\hbar$ determines the rate of the electron capture from the continuous spectrum to any vacant atomic level, following by emission of a photon. In some cases the photon may be absorbed by the nucleus, exciting it. The latter effect, called Nucleus Excitation by Electron Capture (NEEC), was intensively explored in past years. Γ_2 stands for the width Γ_{is} of the nuclear $3/2^+$ isomer. Notice also that the level $|c_2\rangle$ is degenerated over the magnetic quantum numbers. In this case the Green matrix is determined by a system of algebraic equations

$$\left(E_a - E_1 + i\frac{\Gamma_1}{2}\right) G^+_{c_1c_1} - \sum_{c_2} R^+_{c_1c_2} G^+_{c_2c_1} = 1, \; - R^+_{c_2c_1} G^+_{c_1c_1} + \left(E_a - E_2 + i\frac{\Gamma_2}{2}\right) G^+_{c_2c_1} = 0,$$

where the R matrix

$$R^+_{c_1c_2} = \sum_{b' \neq c_1, c_2} \frac{V_{c_1b'} V_{b'c_2}}{E_a + i\eta - E_{b'}} + \dots$$

is responsible for the NEEC process.

The off-diagonal elements of Green's matrix are

$$G^{+}_{c_2c_1} = \frac{R^{+}_{c_2c_1}}{\left(E_a - E_1 + i\frac{\Gamma_1}{2}\right)\left(E_a - E_2 + i\frac{\Gamma_2}{2}\right) - \sum_{c_2}|R^{+}_{c_2c_1}|^2}.$$

The term $\sum_{c_2}|R_{c_2c_1}|^2$ may be eliminated here since it is much less than $\Gamma_{\mathbf{is}}\Gamma_1$. Then the transition matrix takes the form

$$T_{ba} = -\sum_{c_2,c_1}\frac{V'_{bc_2}R^{+}_{c_2c_1}V_{c_1a}}{(\varepsilon - \varepsilon_0 - i\Gamma_1/2)\,(\omega - \omega_{\mathbf{res}} + i\Gamma_{\mathbf{is}}/2)},$$

where the energy $\varepsilon_0 = \hbar^2\kappa_0^2/2m_e = \omega + \epsilon_i$ indicates the maximum of the energy distribution of the conversion electrons,

$$\omega_{\,\mathbf{res}} = \mathbf{E_{is}} + \epsilon_f - \epsilon_i$$

is the resonance value of the laser photon energy.

The matrix element V_{c_1a} determines the first stage of the EB process, caused by absorption of the laser photon by an electron, which initially occupies the level $|j_im_i\rangle$. Applying the multipole expansion of the interaction V_r, one can write the matrix element for the E1 transition as

$$V_{c_1a} = -2\pi i\sqrt{\frac{2\hbar\omega}{3}} \qquad \times \sum_{\mu=-1}^{1} D^1_{\mu\lambda}(\alpha\beta 0)\langle\psi^{+}_{\boldsymbol{\kappa\nu}}|\mathfrak{N}_\mu(E1)|\phi_{j_im_i}\rangle,$$

where $D^1_{\mu\lambda}(\alpha\beta 0)$ is the rotation matrix, depending on the spherical angles β, α of the wave vector k of the photon with polarization $\lambda = \pm 1$.

The electric dipole operator for the electron is

$$\mathfrak{N}_\mu(E1) = -erY_{1\mu}(\hat{\mathbf{r}}) \qquad (e > 0).$$

The R matrix, which determines exchange by virtual photon between two charged particles, has been calculated by Alder *et al.*. For M1 transitions, neglecting penetration of electrons inside the nucleus, one has

$$R^{+}_{c_2c_1} = \frac{4\pi}{3}\sum_{\mu}(-1)^{\mu}\langle I_e M_e|\mathfrak{M}_{\mu}(M1)|I_g M_g\rangle \times \langle j_f m_f|\mathfrak{N}_{-\mu}(M1)|j_c m_c\rangle,$$

where $\mathfrak{M}_{\mu}(M1)$ and $\mathfrak{N}_{\mu}(M1)$ denote the magnetic dipole operators for the nucleus and electron, respectively. In the nonrelativistic approximation for the nucleus

$$\mathfrak{M}_{\mu}(M1) = \mu_n\sqrt{\frac{3}{4\pi}}\sum_{i=1}^{A}\left[g_l(i)l_{\mu}(i) + g_s(i)s_{\mu}(i)\right],$$

where $\mu_n = e\hbar/2m_pc$ is the nuclear magneton, m_p the mass of proton, $l_{\mu}(i)$ and $s_{\mu}(i)$ are the spherical projections of the orbital angular momentum and spin operators for the ith nucleon, $g_l(i)$, $g_s(i)$ the corresponding g-factors.

For the electron

$$\mathfrak{N}_{\mu}(M1) = -\frac{k^2}{2c}\int \mathbf{J}_e\cdot\mathbf{l}\left(h_1^{(1)}(kr)Y_{1\mu}(\hat{\mathbf{r}})\right)d\mathbf{r},$$

where $h_l^{(1)}(x)$ is the spherical Hankel function of the first kind. In the same nonrelativistic (long-wave) approximation this reduces to

$$\mathfrak{N}_{\mu}(M1) = \mu_e\sqrt{\frac{3}{4\pi}}\left[l_{\mu} + 2s_{\mu}\right],$$

where $\mu_e = e\hbar/2m_ec$ is the Bohr magneton for the electron.

Cross section

The cross section of the EB process is expressed by standard formula

$$\sigma_{\mathbf{EB}}(\omega) = \frac{2\pi}{\hbar c}\frac{1}{(2I_g+1)(2j_i+1)}\sum_{M_g,m_i}\sum_{b}|T_{ba}|^2\delta(E_b-E_a).$$

Simple calculations give

$$\sigma_{\mathbf{EB}}(\omega) = (2\pi)^3\frac{2k}{3}\left(\frac{4\pi}{3}\right)^2\frac{\Gamma_{\mathbf{is}}}{(\omega-\omega_{\mathbf{res}})^2+(\Gamma_{\mathbf{is}}/2)^2}\mathcal{F}(\omega),$$

where the designations are

$$\mathcal{F}(\omega) = \frac{1}{(2I_g+1)(2j_i+1)}\sum_{M_e,M_g}\sum_{m_f,m_i}\left|\sum_{\mu\mu'}(-1)^{\mu+\mu'}\langle I_eM_e|\mathfrak{M}_\mu(M1)|I_gM_g\rangle D^1_{\mu'\lambda}(\alpha\beta0)\mathcal{I}_{\mu\mu'}\right|^2,$$

and

$$\mathcal{I}_{\mu\mu'} = \int_0^\infty \frac{f_{\mu\mu'}(\kappa)\kappa^2 d\kappa}{\kappa^2-\kappa_0^2-i\gamma}, \qquad \gamma = m_e\Gamma_1/\hbar^2,$$

with

$$f_{\mu\mu'}(\kappa) = \frac{2m_e}{\hbar^2}\sum_\nu\int d\Omega_{\hat{\boldsymbol{\kappa}}}\langle j_fm_f|\mathfrak{N}_{-\mu}(M1)|\psi^+_{\boldsymbol{\kappa}\nu}\rangle\langle\psi^+_{\boldsymbol{\kappa}\nu}|\mathfrak{N}_{\mu'}(E1)|j_im_i\rangle.$$

It simplifies by employing the expression for $\psi^{+}_{\boldsymbol{\kappa}\nu}$ and using the orthogonality relation:

$$f_{\mu\mu'}(\kappa) = \frac{2m_e}{\hbar^2} \sum_{j_c m_c} \langle j_f m_f | \mathfrak{N}_{-\mu}(M1) | j_c m_c \rangle \langle j_c m_c | \mathfrak{N}_{\mu'}(E1) | j_i m_i \rangle. \tag{1}$$

Further, the integral $\mathcal{I}_{\mu\mu'}$ is transformed to

$$\mathcal{I}_{\mu\mu'} = \frac{1}{2\kappa_0} \int_0^\infty f_{\mu\mu'}(\kappa) \left(\frac{1}{\kappa - \kappa_0 - i\gamma'} - \frac{1}{\kappa + \kappa_0 + i\gamma'} \right) \kappa^2 d\kappa.$$

with the integrand, which may be transformed to a product of smooth function and the sharp factor like a delta-function

$$\frac{1}{\pi} \frac{\gamma'}{(\kappa - \kappa_0)^2 + \gamma'^2} \approx \delta(\kappa - \kappa_0).$$

Trivial estimation gives

$$\mathcal{I}_{\mu\mu'} = \pi i \rho(\varepsilon_0) \sum_{j_c m_c} \langle j_f m_f | \mathfrak{N}_{-\mu}(M1) | j_c m_c \rangle \langle j_c m_c | \mathfrak{N}_{\mu'}(E1) | j_i m_i \rangle,$$

where

$$\rho(\varepsilon_0) = \frac{\kappa m_e}{\hbar^2}$$

is the density of the electron states.

The $\mathfrak{M}_\mu(\lambda l)$ and $\mathfrak{N}_\mu(\lambda l)$, where $\lambda = E$ or M, are the irreducible tensor operators.

We apply the Wigner-Eckart theorem

$$\langle j_f m_f|\mathfrak{M}_\mu(\lambda l)|j_i m_i\rangle = \frac{(j_i l m_i \mu|j_f m_f)}{\sqrt{2j_f+1}}\langle j_f||\mathfrak{M}(\lambda l)||j_i\rangle,$$

where $\langle j_f||\mathfrak{M}(\lambda l)||j_i\rangle$ is the reduced matrix element, related to the reduced transition probability by

$$B(\lambda l; i \to f) = (2j_i+1)^{-1}|\langle j_f||\mathfrak{M}(\lambda l)||j_i\rangle|^2.$$

Using orthogonality rule for the Clebsh-Gordan coefficients

$$\sum_{m_1 m}(j_1 j_2 m_1 m_2|jm)(j_1 j_2' m_1 m_2'|jm) = \frac{2j+1}{2j_2+1}\delta_{j_2 j_2'}\delta_{m_2 m_2'},$$

unitarity of the Wigner functions

$$\sum_{\mu=-j}^{j} D^{j*}_{\mu\nu}(\alpha\beta\gamma)D^{j}_{\mu\nu'}(\alpha\beta\gamma) = \delta_{\nu\nu'},$$

the connection of the reduced probabilities for direct and reverse transitions

$$\frac{B(\lambda l; i \to f)}{B(\lambda l; f \to i)} = \frac{2j_f+1}{2j_i+1},$$

and

$$\sum_{\nu}\int d\Omega_{\boldsymbol{\kappa}}\mathfrak{Y}^{m_c'*}_{j_c'}(l' s\nu;\hat{\boldsymbol{\kappa}})\mathfrak{Y}^{m_c}_{j_c}(ls\nu;\hat{\boldsymbol{\kappa}}) = \ = \delta_{ll'}\delta_{j_c j_c'}\delta_{m_c m_c'}.$$

one arrives at

$$\mathcal{F}(\omega) = \rho^2(\varepsilon_0)\left(\frac{\pi}{3}\right)^2 g_{j_c} B_n(M1; I_e \to I_g) \sum_{j_c} B_e(M1; j_f \to j_c) B_e(E1; j_i \to j_c),$$

where the reduced probabilities B_n and B_e refer to the nuclear and electronic transitions, respectively, and the spin factor

$$g_{j_c} = \frac{2I_e + 1}{2I_g + 1} \frac{2j_f + 1}{2j_c + 1}.$$

For the EB, shown in Fig.1a, the intermediate electron state $|c\rangle$ is a coherent mixture of the waves with angular momenta $j_c = 1/2$ and $j_c = 3/2$, while for the EB in Fig.1b it reduces to single wave with $j_c = 1/2$.

Substitution of (??) into (??) completely determines the EB cross section:

$$\sigma_{\mathbf{EB}}(\omega) = \sum_{\mathbf{j_c}} \sigma_{\mathbf{EB}}^{\mathbf{j_c}}(\omega),$$

where the partial cross section for the branching transition via the electron state j_c with the positive energy ε is

$$\sigma_{\mathbf{EB}}^{\mathbf{j_c}}(\omega) = \frac{\mathbf{1}}{\mathbf{8}} \mathbf{g_{j_c}} \frac{\mathbf{\Gamma_{is}} \mathbf{\Gamma_{IC}^{j_c}(f)}}{(\omega - \omega_{\mathbf{res}})^{\mathbf{2}} + (\mathbf{\Gamma_{is}/2})^{\mathbf{2}}} \sigma_{\mathbf{ion}}^{\mathbf{j_c}}(\omega).$$

Besides, here $\sigma_{\rm ion}^{j_c}(\omega)$ is the partial ionization cross section for an atomic electron, occupying initially the j_i shell and carrying away the total angular momentum j_c:

$$\sigma_{\rm ion}^{j_c}(\omega) = (2\pi)^4 \frac{4k}{9} B_e(E1; j_i \to j_c)\rho(\varepsilon_0),$$

while the complete ionization cross section is

$$\sigma_{\rm ion}(\omega) = \sum_{j_c} \sigma_{\rm ion}^{j_c}(\omega).$$

The partial width for the internal electron conversion through transition $j_f \to j_c$ is given by

$$\Gamma_{\rm IC}^{j_c}(f) = 4\left(\frac{2\pi}{3}\right)^3 B_n(M1; I_e \to I_g) B_e(M1; j_f \to j_c)\rho(\varepsilon_0)$$

and complete width $\Gamma_{\rm IC}$ is a sum of $\Gamma_{\rm IC}^{j_c}$.

For the transition $(a) = \{s \to p \to p\}$ the intermediate electron state $|c\rangle$ is a coherent mixture of the waves with angular momenta $j_c = 1/2$ and $j_c = 3/2$. On the electron path $(b) = \{p \to s \to s\}$ it reduces to single wave with $j_c = 1/2$. In the first case

$$B_e(E1; 1/2 \to 3/2) = 2B_e(E1; 1/2 \to 1/2).$$

Respectively, the partial ionization cross sections are

$$\sigma_{\rm ion}^{3/2}(\omega) = \frac{2}{3}\sigma_{\rm ion}(\omega), \qquad \sigma_{\rm ion}^{1/2}(\omega) = \frac{1}{3}\sigma_{\rm ion}(\omega).$$

Then one can rewrite the EB cross section for the transition (a) as

$$\sigma_{\rm EB}^{(a)}(\omega) = \frac{1}{24} \frac{\Gamma_{\rm is}(2g_{3/2}\Gamma_{\rm IC}^{3/2} + g_{1/2}\Gamma_{\rm IC}^{1/2})}{(\omega - \omega_{\rm res})^2 + (\Gamma_{\rm is}/2)^2} \sigma_{\rm ion}(\omega). \tag{2}$$

Averaging of the cross section

The radiation emitted by a laser source is characterized by some energy distribution normalized to unity:

$$\int_0^\infty w_s(\omega)d\omega = 1.$$

The experimentally measured cross section is

$$\langle \sigma_{\mathbf{ion}}^{j_c}(\delta) \rangle = \int_{-\infty}^{\infty} d\omega w_s(\omega) \sigma_{\mathbf{ion}}^{j_c}(\omega),$$

where δ is a detuning of the incident laser pulse from the resonance,

$$\delta = \omega_{\mathbf{res}} - \omega_0.$$

I take the Lorentzian distribution

$$w_s(\omega) = \frac{1}{2\pi}\frac{\Gamma_e}{(\omega-\omega_0)^2+(\Gamma_s/2)^2},$$

Inserting its integral representation

$$w_s(\omega) = \frac{1}{\pi}\mathbf{Re}\int_0^\infty d\mu e^{i(\omega-\omega_0)\mu-\Gamma_s\mu/2}$$

and applying the contour integration one finds

$$\langle\sigma^{j_c}_{\mathbf{EB}}(\delta)\rangle = \langle\sigma^{j_c}_{\mathbf{EB}}(0)\rangle\frac{(\Gamma/2)^2}{\delta^2+(\Gamma/2)^2},$$

where $\Gamma = \Gamma_s + \Gamma_{\mathbf{is}}$ and the resonant value of the averaged cross section is

$$\langle\sigma^{j_c}_{\mathbf{EB}}(0)\rangle = \frac{1}{2}g_{j_c}\frac{\Gamma^{j_c}_{\mathbf{IC}}}{\Gamma}\sigma^{j_c}_{\mathbf{ion}}(\omega_{\mathbf{res}}).$$

The isomer ^{229m}Th mainly decays through the internal conversion channel ($\alpha \approx 10^9$), so that $\Gamma_{\mathbf{IC}}(7s) \approx \Gamma_{\mathbf{is}}$. Therefore the resonant value of the cross section, corresponding to the path (b), will be

$$\langle \sigma_{\mathbf{EB}}(0) \rangle_b \approx \frac{1}{2} \left(\frac{2I_e + 1}{2I_g + 1} \right) \frac{\Gamma_{\mathbf{is}}}{\Gamma_s + \Gamma_{\mathbf{is}}} \sigma_{\mathbf{ion}}(\omega_{\mathbf{res}}).$$

Let us compare it with the absorption cross section of ultraviolet photons

$$\langle\sigma_\gamma(\delta)\rangle = \langle\sigma_\gamma(0)\rangle\frac{(\Gamma/2)^2}{\delta^2+(\Gamma/2)^2},$$

where the cross section at the resonance

$$\langle\sigma_\gamma(0)\rangle = \frac{2\pi}{k_0^2}\left(\frac{2I_e+1}{2I_g+1}\right)\frac{\Gamma^\gamma_{\mathbf{is}}}{\Gamma}.$$

Here $k_0=\omega_0/c$ and $\Gamma^\gamma_{\mathbf{is}}$ is the partial radiation width of the isomer

$$\Gamma^\gamma_{\mathbf{is}} = \Gamma_{\mathbf{is}}/(1+\alpha) \approx 10^{-9}\Gamma_{\mathbf{is}}).$$

Taking the value $\sigma_{\mathbf{ion}}(8.3\,\mathbf{eV}) \approx 3\cdot 10^{-18}$ $\mathbf{cm}^2$, calculated by Borisyuk, one gets the estimation

$$\langle\sigma_\gamma(0)\rangle/\langle\sigma_{\mathbf{IB}}(0)\rangle_b \approx 0.02$$

In the case (a) the cross section $\langle\sigma^{(a)}_{\mathbf{EB}}(0)\rangle$ is by order of magnitude less than $\langle\sigma_{\mathbf{EB}}(0)\rangle_b$.

Advantages of the method

As a whole, my calculations confirm the conclusion of the paper [?] that EB via continuum, and especially its version (b), is more perspective than the direct photoexcitation of the isomer ^{229m}Th. The EB via the continuum has also an advantage compared to the EB via discrete atomic levels (see also Ref. [?]). In the last case we have to realize two resonances. At first, the laser frequency ω should match the transition frequency in the atom ω_a, i.e. it should be

$$\omega \approx \omega_a = (\epsilon_c - \epsilon_i)/\hbar,$$

where ϵ_i, ϵ_c and ϵ_f are the energies of initial, intermediate and final atomic levels. Furthermore, we have to provide one more constraint

$$\omega \approx (E_{\text{is}} + \epsilon_f - \epsilon_i)/\hbar.$$

Comparing these two requirements we get the main resonance condition for the atomic and nuclear transitions $E_{\text{is}} \approx \epsilon_c - \epsilon_f$, that ensures effective transfer of the atomic energy to the nucleus. Note that the same resonance condition is needed for nuclear excitation by electron transition (NEET), However, it is difficult to find such atomic transitions between bound levels, which are almost in the resonance with the nuclear transition. At the same time, in the case of the continuum EB two resonance conditions, which are reduced to single one $\omega \approx \omega_{\text{res}}$, are always fulfilled automatically.

As a benchmark of the population of ^{229m}Th one can employ the products of its decay - the conversion electrons. The prompt photoelectrons are emitted during the action of the laser pulse, while the delayed conversion electrons are ejected during the isomer decay. These electrons are easily separated in the time-delay experiment by choosing duration of the laser pulse $\tau_s = \hbar/\Gamma_s$ much larger than the isomer lifetime $\tau_{\text{is}} = \hbar/\Gamma_{\text{is}}$. However, this corresponds to wide frequency distribution of the laser photons, $\Gamma_s >> \Gamma_{\text{is}}$ and respectively to weakening of the counting rate of successful events.

Thanks for attention